Abstract Book

The 8th Young Scholar Symposium, East Asia Section of Inverse Problems International Association

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Sponsor: School of Mathematics and Statistics, Central South University

Plenary speakers:

Hongyu Liu, City University of Hong Kong (hongyliu@cityu.edu.hk)

Gen Nakamura, Hokkaido University (gnaka@math.sci.hokudai.ac.jp)

List of young speakers:

1. Xinlin Cao, The Hong Kong Polytechnic University (xinlin.cao@polyu.edu.hk)
2. Guozhi Dong, Central South University (guozhi.dong@csu.edu.cn)
3. Shota Fukushima, Chiba Institute of Technology (fukushima.shota@p.chibakoudai.jp)
4. Kuang Huang, The Chinese University of Hong Kong (kuanghuang@cuhk.edu.hk)
5. Yong-Gwan Ji, Korea Institute for Advanced Study (ygji@kias.re.kr)
6. Sangwoo Kang, Pusan National University (sangwoo.kang@pusan.ac.kr)
7. Jaeyong Lee, Chung-Ang University (jaeyong@cau.ac.kr)
8. Haibo Li, The University of Melbourne (haibo.li@unimelb.edu.au)
9. Hongjie Li, Tsinghua University (hongjieli@tsinghua.edu.cn)
10. Jianliang Li, Hunan Normal University (lijianliang@hunnu.edu.cn)
11. Yonglin Li, Wuhan University (yonglin.li@whu.edu.cn)
12. Hao Liu, Hong Kong Baptist University (haoliu@hkbu.edu.hk)
13. Ping Liu, Zhejiang University (pingliu@zju.edu.cn)
14. ~~Shiqi Ma, Jilin University (mashiqi@jlu.edu.cn)~~ cancelled
15. Lingyun Qiu, Tsinghua University (lyqiu@tsinghua.edu.cn)
16. Qingxiang Shi, Tsinghua University (sqxsqx142857@tsinghua.edu.cn)
17. Hiroshi Takase, Kyushu University (htakase@imi.kyushu-u.ac.jp)

Optical inversion using plasmonic contrast agents

Xinlin Cao (The Hong Kong Polytechnic University, xinlin.cao@polyu.edu.hk)

In this talk, we introduce a new method to reconstruct the permittivity distribution, of an object to image, from the remotely measured electromagnetic field. We propose to use the remote fields measured before and after injecting locally in the medium plasmonic nano-particles. Such a technique is known in the framework of imaging using contrast agents where, in optical imaging, the nano-particles play the role of these contrast agents.

Second-order dynamics, an old bottle of new wine

Guozhi Dong (Central South University, guozhi.dong@csu.edu.cn)

Second-order dynamics is a classical model in mechanics to describe a conversion and dissipation of energy in a mechanic system. In this talk we present that it can also be a powerful tool to help developing numerical methods in inverse problems and scientific computing in general. I will show cases by selecting some specific examples.

Surface vector field decomposition and topological characterization of codimensions

Shota Fukushima (Chiba Institute of Technology, fukushima.shota@p.chibakoudai.jp)

In three- or higher- dimensional space, any vector field defined on a boundary of smooth bounded domain can be decomposed into three components based on the properties of its harmonic extensions to inside and outside the domain. This decomposition is not direct in general and the codimension of this non-directness is characterized by the first Betti number of the boundary. Each component of the decomposition corresponds to each essential spectrum of the elastic Neumann-Poincaré operator, which has become important recently in relation to metamaterials. This talk is based on joint works with Yong-Gwan Ji (Korea Institute for Advanced Study) and Hyeonbae Kang (Inha University).

Inverse source problem in a nonlocal traffic flow model for network trip flows

Kuang Huang (The Chinese University of Hong Kong, kuanghuang@cuhk.edu.hk)

This talk will introduce a nonlocal transport equation for modeling network trip flows of privately operated vehicles in road networks. Along with the well-posedness of the direct problem, we investigate an inverse source problem with parameters reflecting specific traffic scenarios. We establish conditional Lipschitz stability under suitable assumptions and present a user-friendly numerical method for reconstructing flow rates, along with an error analysis.

Finiteness of the stress in presence of closely located inclusions with imperfect bonding

Yong-Gwan Ji (Korea Institute For Advanced Study, ygji@kias.re.kr)

If two conducting or insulating inclusions are closely located, the gradient of the solution may become arbitrarily large as the distance between inclusions tends to zero, resulting in high concentration of stress in between two inclusions. This happens if the bonding of the inclusions and the matrix is perfect, meaning that the potential and flux are continuous across the interface. In this paper, we consider the case when the bonding is imperfect. We consider the case when there are two circular inclusions of the same radii with the imperfect bonding interfaces and prove that the gradient of the solution is bounded regardless of the distance between inclusions if the bonding parameter is finite. This result is of particular importance since the imperfect bonding interface condition is an approximation of the membrane structure of biological inclusions such as biological cells. This is joint work with Shota Fukushima, Hyeonbae Kang, and Xiaofei Li.

Sampling-type imaging methods for inverse scattering problem in various measurement configurations

Sangwoo Kang (Pusan National University, sangwoo.kang@pusan.ac.kr)

The development and analysis of efficient methods and techniques for solving an inverse scattering problem are of great interest due to their potential in various applications, such as nondestructive testing, biomedical imaging, radar imaging, and structural imaging, among others. Sampling-type imaging methods allow us to non-iteratively retrieve the support of (possibly multiconnected) targets with low computational cost, assuming no a priori information about the targets. A sampling method tests a region of interest with its associated indicator function; the indicator function blows up if a test location is in support of inhomogeneities. Even though the sampling methods show promising results in ideal(multistatic, full-aperture, sufficiently many receivers) measurement configuration, one can frequently encounter limited measurement cases in practical applications. This presentation introduces the sampling-type imaging methods in two-dimensional limitedaperture, monostatic, and bistatic measurement cases. We identify the asymptotic structure of imaging methods to explore the applicability and intrinsic properties.

Deep learning approaches for solving forward and inverse problems in partial differential equations

Jaeyong Lee (Chung-Ang University, jaeyong@cau.ac.kr)

Partial differential equations (PDEs) are fundamental for modeling a wide range of physical phenomena. Solving both forward and inverse problems in PDEs has become crucial in many scientific and engineering fields. Recently, deep learning has emerged as a powerful tool for addressing these challenges. Two primary deep learning approaches are commonly used to approximate PDE solutions: using neural networks to directly parametrize the solution and learning operators that map PDE parameters to their solutions. One approach is the Physics-Informed Neural Network (PINN) introduced by Raissi, Perdikaris, and Karniadakis (2019), which learns neural network parameters by minimizing the PDE residuals in a least-squares sense. Alternatively, operator learning via neural networks has been explored to approximate the complex, nonlinear solution operators of PDEs. In this talk, I will present these two methods for solving PDEs and discuss my research related to them.

Scalable iterative data-adaptive RKHS regularization

Haibo Li (The University of Melbourne, haibo.li@unimelb.edu.au)

We present iDARR, a scalable iterative Data-Adaptive RKHS Regularization method, for solving ill-posed linear inverse problems. The method searches for solutions in subspaces where the true solution can be identified, with the data-adaptive RKHS penalizing the spaces of small singular values. At the core of the method is a new generalized Golub-Kahan bidiagonalization procedure that recursively constructs orthonormal bases for a sequence of RKHS-restricted Krylov subspaces. The method is scalable with a complexity of O(kmn) for m-by-n matrices with k denoting the iteration numbers. Numerical tests on the Fredholm integral equation and 2D image deblurring show that it outperforms the widely used L2 and l2 norms, producing stable accurate solutions consistently converging when the noise level decays.

The effective construction on elastic metamaterials

Hongjie Li (Tsinghua University, hongjieli@tsinghua.edu.cn)

In this talk, we explore various structures designed to achieve negative elastic metamaterials. First, we review the effective methods for constructing negative mass density and bulk modulus. Next, we focus on the strategies for achieving a negative shear modulus.

Inverse random potential scattering for the polyharmonic wave equation: far-field patterns

Jianliang Li (Hunan Normal University, lijl@amss.ac.cn)

Consider the inverse random potential scattering problem for the two- and three-dimensional polyharmonic wave equation. The potential is modeled as a centered, complex-valued generalized microlocally isotropic Gaussian (GMIG) random field with its covariance and relation operators being classical pseudo-differential operators. Two uniqueness results are obtained. First, when the real and imaginary parts of the random potential have the same order, we demonstrate that, with probability one, the principal symbols of the covariance and relation operators can be uniquely determined by the backscattering far-field data averaged over the frequency band, generated by a single realization of the random potential. Second, if the real and imaginary parts have the different orders, a single realization of the backscattering far-field data averaged over the frequency band can uniquely recover the principal symbol of the covariance operator for the part with the higher order. The analysis employs the Born approximation in the high frequency regime, the microlocal analysis for the Fourier integral operators, and the ergodicity of the wave field.

High-order convergent finite element methods for linear elasticity problems with close-to-touching inclusions

Yonglin Li (Wuhan University, yonglin.li@whu.edu.cn)

To numerically solve the linear elasticity problem with partially infinite Lame parameters and closely located inclusions, we develop high-order vectorial finite element methods on two cases of graded meshes based on the asymptotic estimates. For properly separated inclusions, we prove that the proposed method achieves optimal convergence rates for arbitrarily high orders. In cases of nearly touching inclusions, the method remains optimal for p=1,2. Extensive numerical experiments in two and three dimensions are presented to demonstrate the accuracy of the numerical solutions. Additionally, the blow-up behaviours of the gradients of solutions are observed when the inclusions approach each other.

Exploiting low-dimensional data structures by deep neural networks with applications in operator learning

Hao Liu (Hong Kong Baptist University, haoliu@hkbu.edu.hk)

Deep neural networks have demonstrated a great success in many applications, especially for problems with high-dimensional data sets. In spite of that, most existing statistical theories are cursed by data dimension and cannot explain such a success. To bridge the gap between theories and practice, we exploit the low-dimensional structures of data set and establish theoretical guarantees with a fast rate that is only cursed by the intrinsic dimension of the data set. Autoencoder is a powerful tool in exploring data low-dimensional structures. In our work, we analyze the approximation error and generalization error of autoencoder and its application in operator learning. Our results provide fast rates depending on the intrinsic dimension of data sets and show that deep neural networks are adaptive to low-dimensional structures of data sets.

Inverse problem for large population dynamics

Hongyu Liu (City University of Hong Kong, hongyliu@cityu.edu.hk)

A famous quote says: "The most incomprehensible thing about the world is that it is comprehensible." Following this quote, the most incomprehensible thing about biological species, including human beings, is that they are comprehensible, but in the large scale. Hence, they can be quantitatively understood, predicted and even controlled, which naturally leads to various inverse problems. In this talk, I will discuss our recent study of inferring the large population dynamics from observation of the corresponding dynamical behaviours with applications. Emphasis will be given to inverse problems for several biological models and mean field games.

A mathematical theory of computational resolution limit and super-resolution

Ping Liu (Zhejiang University, pingliu@zju.edu.cn)

Due to the physical nature of wave propagation and diffraction, there is a fundamental diffraction barrier in optical imaging systems which is called the diffraction limit or resolution limit. Rayleigh investigated this problem and formulated the well-known Rayleigh limit. However, the Rayleigh limit is empirical and only considers the resolving ability of the human visual system. On the other hand, resolving sources separated below the Rayleigh limit to achieve so-called "super-resolution" has been demonstrated in many numerical experiments.

In this talk, we will propose a new concept "computational resolution limit" which reveals the fundamental limits in super-resolving the number and locations of point sources from a data-processing point of view. We will quantitatively characterize the computational resolution limits by the signal-to-noise ratio, the sparsity of sources, and the cutoff frequency of the imaging system. As a direct consequence, it is demonstrated that l_0 optimization achieves the optimal order resolution in solving super-resolution problems. For the case of resolving two point sources, the resolution estimate is improved to an exact formula. We will also propose an optimal algorithm to distinguish images generated by single or two point sources. Generalization of our results to the imaging of positive sources and imaging in multi-dimensional spaces will be briefly discussed as well.

Participation cancelled

Shiqi Ma (Jilin University, mashiqi@jlu.edu.cn)

Cancelled

Analysis on anisotropic spring-dashpot models and their Boltzmann type viscoelastic systems

Gen Nakamura (Hokkaido University, gnaka@math.sci.hokudai.ac.jp)

Spring-dashpot models (abbreviated by SDM) and Boltzmann-type viscoelastic system of equations (abbreviated by BVS) are two major types of systems of equations for viscoelasticity. The convolution kernel of the stress-strain relation for the SDM is called the relaxation tensor. If the relaxation tensor is derived, the SDM can be converted to the BVS. Concerning the relaxation tensor, the typical SDM are the Maxwell model, Burgers model, standard linear solid model, and their extended versions. By measuring the relaxation time, we can have qualitative information on the relaxation tensor.

The derivation of the relaxation tensor for the anisotropic extended Burgers model has not been known. In the first part of my talk, I will show how to derive the relaxation tensor for this model and analyze its properties (see [5]). Further, I will show the exponential decay of solutions of the initial boundary value problem with the mixed-type boundary condition for the BVS (see [2]).

The second part of my talk is about the control theory for the SDM. Since the BVS is an integrodifferential system of equations, the solutions of the BVS do not generate semigroup. This is one of the major reasons that it is hard to study the control theory for the BVS. While, for those of the extended Maxwell model, I will show in the second part of my talk that we can have the generation of group (see [4]). Based on this, I will discuss the control theory for the extended Maxwell model(see [3]).

If there is some time left, I will touch on an inverse problem for the BVS as the continuation of my study giving the foundation for the vibroseis reflection exploration of anisotropic viscoelastic grounds when their physical properties are either piecewise homogeneous or piecewise isotropic (see [1], [6], [7]).

References

[1] C. Carstea, G. Nakamura and L. Oksanen, Uniqueness for the inverse boundary value problem of piecewise homogenous anisotropic elasticity in the time domain, Trans. AMS.,373 (2020) pp.3423-3443.

[2] M. de Hoop, C-L. Lin and G. Nakamura, Uniform decaying property of solutions for anisotropic viscoelastic systems, arXiv 2308.03988 (2023).

[3] M. de Hoop, C-L. Lin and G. Nakamura, EXact boundary controllability for reduced system associated to extended Maxwell system, arXiv: 2408.07274 (2024).

[4] M. de Hoop, M. Kimura, C-L. Lin and G. Nakamura, Resolvent estimates for viscoelastic systems of extended Maxwell type and their applications, arXiv 2308.1622v (2023) to appear in SIAM J. Math. Analysis.

[5] M. de Hoop, M. Kimura, C-L. Lin, G. Nakamura and K. Tanuma, Anisotropic extended Burgers model, its relaxation tensor and properties of the associated Boltzmann viscoelastic system, arXiv:2406.18978 (2024).

[6] M. de Hoop, G. Nakamura and J. Zhai, Unique recovery of piecewise analytic density and stiffness tensors from the elasti-wave Dirichlet-to-Neumann map, SIAM J. Appl. Math.,79 (2019) https://doi.org/10.1137/18M1232802

[7] M. Eller, N. Honda, C-L. Lin and G. Nakamura, Global unique continuation from the boundary for a system of viscoelasticity with analytic coefficients and a memory term, Inverse Problems and Imaging, 16 (2022) pp. 1529-1542.

Sediment measurement: an inverse problem formulation

Lingyun Qiu(Tsinghua University, `lyqiu@tsinghua.edu.cn`)

In this work, we present a novel approach for sediment concentration measurement in water flow, modeled as a multiscale inverse medium problem. To address the multiscale nature of the sediment distribution, we treat it as an inhomogeneous random field and use the homogenization theory in deriving the effective medium model. The inverse problem is formulated as the reconstruction of the effective medium model, specifically, the sediment concentration, from partial boundary measurements. Additionally, we develop numerical algorithms to improve the efficiency and accuracy of solving this inverse problem. Our numerical experiments demonstrate the effectiveness of the proposed model and methods in producing accurate sediment concentration estimates, offering new insights into sediment measurement in complex environments.

Novel sampling methods for acoustic sources from multi-frequency far field patterns at sparse observation directions

Qingxiang Shi (Tsinghua University, sqxsqx142857@tsinghua.edu.cn)

This work is dedicated to novel uniqueness results and high resolution sampling methods for source support from multi-frequency sparse far field patterns. If the source support is composed of polygons and annuluses, then we prove that the support can be determined by multi-frequency far field patterns at sparse directions. Precisely, the lowest number of the observation directions is given in terms of the number of the corners and the annuluses. Inspired by the uniqueness arguments, we introduce two novel indicators to determine the source support. Numerical examples in two dimensions are presented to show the validity and robustness of the two indicators for reconstructing the boundaries of the source support with a high resolution. The second indicator also shows its powerful ability to determine the unknown source function. Furthermore, these methods can also be applied to reconstruct obstacles and medium.

Lipschitz stability for Cauchy problems of elliptic equations and applications to evolution equations

Hiroshi Takase (Kyushu University, htakase@imi.kyushu-u.ac.jp)

The Cauchy problem for elliptic equations is widely known as an ill-posed problem without stability properties. To overcome this situation, conditional stability has been studied, but in many cases, only very weak logarithmic stability is obtained assuming a priori boundedness of solutions. In this presentation, we show Lipschitz stability estimate for this problem under a new priori constraint. We also see that these methods are applicable to parabolic and hyperbolic equations. This work is based on joint research with Mourad Choulli.